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# Erratum to: "Random walks on weighted, oriented percolation clusters"

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**Abstract.** It was pointed out to me by Matthias Birkner and Sebastian Steiber that the convergence in Equation (4.3) as $m \to \infty$ may not hold, since $Z_n = Z_n(m)$ is a function of $m$ and not constant. In fact, the convergence cannot hold, as already the fourth moment $\tilde{\mathbb{E}}[Z_0^4(m)]$ grows too fast in $m$ to be compensated by the mixing coefficients, as can be seen by comparison with an i.i.d. sequence. As a consequence the annealed central limit Theorem (aCLT), Theorem 1.3, does not hold in the full generality claimed. The limit law is only non-degenerate for weights $K$, which are $\phi$-mixing with coefficients in $\mathcal{O}(n^{-(2+\delta)})$ for any $\delta > 0$. Thus the correct result is as follows.

**Theorem 1.3** (Annealed CLT for polynomially time-mixing weights)**.** *Let $d \geq 1$ and $p \in (p_c, 1)$. If $K$ is independent of $\omega$, strictly positive, stationary and $\phi$-mixing in the time coordinate with mixing coefficients $\phi_n \in \mathcal{O}(n^{-(2+\delta)})$ for some $\delta > 0$, then an aCLT holds, i.e. for all continuous and bounded functions $f \in C_b(\mathbb{R}^d)$*

$$\tilde{\mathbb{E}}\left[f\left(\frac{(X_n - n\vec{\mu})}{\sqrt{n}}\right)\right] \xrightarrow{n\to\infty} \Phi(f), \tag{1.1}$$

*where $\vec{\mu}$ is the same drift vector as in Lemma (1.2), $\Phi(f) := \int f(x)\Phi(\mathrm{d}x)$ and $\Phi$ is a non-trivial centred $d$-dimensional Gaussian law with full rank covariance matrix $\Sigma$.*

The proof for the corrected statement needs a slightly stronger statement in Lemma (3.1). The process $(Y_n, \tau_n)_{n\in\mathbb{N}}$ is $\phi$-mixing (and not only $\alpha$-mixing) with respect to the law $\tilde{\mathbb{P}}$.

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**Lemma 3.1** (Increments of the random walk are ergodic)**.** *Let $d \geq 1$, $K$ be independent of $\omega$, stationary and $\phi$-mixing in the time coordinate with mixing coefficients $(\phi_n)_{n\in\mathbb{N}}$. Then the process $(Y_n, \tau_n)_{n\in\mathbb{N}}$ is stationary and $\phi$-mixing with respect to $\tilde{\mathbb{P}}$ with mixing coefficients*

$$(\phi_n^X)_{n\in\mathbb{N}} = (\phi_{2mn} + 2\alpha_{2mn}^P)_{n\in\mathbb{N}}, \tag{3.2}$$

*where $\alpha_n^P = Ce^{-cn}$, $n \in \mathbb{N}$, are the mixing coefficients for $\xi^P$ from Lemma (2.1), Equation (2.2).*

**Proof**

*Proof of Lemma 3.1:* The proof requires only slight modifications from the proof of old Lemma (3.1) to strengthen the result to the new claim. Denote by $\mathcal{W}$ the $\sigma$-algebra that contains all possible paths of the random walk, namely

$$\mathcal{W}_k^l := \sigma\left(\{(X_i(\omega), i)\}_{i=k}^l : \omega \in \Omega\right)$$

and $\mathcal{W} = \mathcal{W}_0^\infty$. Define the $\phi$-mixing coefficients for the process $(X_{T_n} - X_{T_{n-1}})_{n\in\mathbb{N}}$ similar to Equation (3.12)

$$\phi_n^X = \sup_{N\in\mathbb{N}} \sup_{\substack{W\in\mathcal{W}, \\ A^N := W\cap\mathcal{W}_0^{T_N}, \\ B^N := W\cap\mathcal{W}_{T_{N+n}}^\infty}} \left| \frac{\tilde{\mathbb{P}}\left(B^N \cap A^N\right)}{\tilde{\mathbb{P}}\left(A^N\right)} - \tilde{\mathbb{P}}\left(B^N\right) \right|.$$

We have to show that the error terms still converge if we divide by $\tilde{\mathbb{P}}(A^N) = \mathbb{P}(A^N)$. The necessary estimate uses independence of the events $A_{(x,l)}^N$ and $\{\xi_l^P(x) = 1\}$ such that

$$\frac{\sum_{(x,l)} \mathbb{P}\left(A_{(x,l)}^N\right)}{\mathbb{P}\left(A^N\right)} = \frac{\sum_{(x,l)} \mathbb{P}\left(A_{(x,l)}^N \cap \{\xi_l^P(x) = 1\}\right)}{\mathbb{P}\left(A^N\right)\mathbb{P}\left(\xi_l^P(x) = 1\right)} = \frac{1}{\mathbb{P}(B_0)}.$$

This leads to new error bounds

$$\mathcal{E}_1 \leq \frac{1}{\mathbb{P}(B_0)} \frac{\sum_{(x,l)} \mathbb{P}(A_{(x,l)}^N)\phi_{2mn}}{\mathbb{P}(A^N)} \leq \frac{1}{\mathbb{P}(B_0)^2}\phi_{2mn}.$$

to replace Equation (3.14) and

$$\mathcal{E}_2 \leq \frac{1}{\mathbb{P}(B_0)} \frac{\sum_{(x,l)} \mathbb{P}(A_{(x,l)}^N)\alpha_{2mn}^P}{\mathbb{P}(A^N)} \leq \frac{1}{\mathbb{P}(B_0)^2}\alpha_{2mn}^P.$$

to replace Equation (3.18). □

*Proof of Theorem 1.3:* We need to show that the variance is strictly positive under the new assumptions. All other parts of the proof remain unchanged. We can now use Theorem 17.2.3 from Ibragimov and Linnik (1971) for uniformly mixing sequences, which gives

$$\begin{aligned} \left|\sum_{n=1}^\infty \tilde{\mathbb{E}}[Z_0 Z_n]\right| &\leq 2\sum_{n=1}^\infty \tilde{\mathbb{E}}[Z_0^2]^{1/2}\tilde{\mathbb{E}}[Z_n^2]^{1/2}(\phi_n^X)^{1/2} \\ &= 2\tilde{\mathbb{E}}[Z_0^2]\sum_{n=1}^\infty (\phi_n^X)^{1/2}. \end{aligned}$$

The sum is finite, since the weights $K$ are $\phi$-mixing with coefficients in $\mathcal{O}(n^{-(2+\delta)})$ for any $\delta > 0$. Furthermore, the sum can be made arbitrary small if we choose $m$ large enough. Choose $m$ so that $|\sum_{n=1}^{\infty}(\phi_n^X)^{1/2}| < 1/4$. Then the variance is strictly positive,

$$\sigma^2 = \tilde{\mathbb{E}}[Z_0^2] + 2\sum_{n=1}^{\infty}\tilde{\mathbb{E}}[Z_0 Z_n] \geq \tilde{\mathbb{E}}[Z_0^2]\left(1 - 4\sum_{n=1}^{\infty}(\phi_n^X)^{1/2}\right) > 0.$$

In this new estimate the term $\tilde{\mathbb{E}}[Z_0^2]$ is still a function of $m$, but it appears as a factor and we only need to make sure that $\tilde{\mathbb{E}}[Z_0^2(m)] > 0$ for any finite $m \in \mathbb{N}$, which is true since the increments are not deterministic. The claim follows from the CLT for uniformly mixing sequences, Theorem 18.5.2 from Ibragimov and Linnik (1971). □

## References

I. A. Ibragimov and Y. V. Linnik. *Independent and stationary sequences of random variables*. Wolters-Noordhoff Publishing, Groningen (1971). MR0322926.